February 19, 2019
Time: 55 minutes
Spring 2018-19

MATHEMATICS 218
QUIZ 1

Circle your section number :

Michella Bou-Eid **Lecture 1** (T, Th at 11:00)	Michella Bou-Eid **Lecture 2** (T, Th at12:30)	Rana Nassif **Lecture 3** (MWF 2 pm)	Hazar Abu-Khuzam **Lecture 4** MWF at 10 am	Sabine El Khoury **Lecture 5** MWF at 9 am

PROBLEM	GRADE
PART I	
1	/ 13
2	/ 13
3	/ 13
4	/ 7
PART II	

5	6	7	8	9	10	11	12

5-12	/ 40
PART III	
13-19	/ 14
TOTAL	/ 100

PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. Find the values of k for which the system

$$\begin{aligned} x + y + z &= 0 \\ x + ky + z &= 1 \\ kx + y + z &= k+1 \end{aligned}$$

has

a. no solution
b. a unique solution
c. infinitely many solutions.

[13 points]

2. Suppose that A is a 3×3 matrix such that

$$(A+I)^{-1}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix},$$

(a) Find det(A).

(b) Show that A is nilpotent ($A^k=\mathbf{0}$ for some positive integer k)

[4 points]

3. (a) If A and B are 3x3 matrices with $\det(2A^{-1}) = 4 = \det(A^{3}(B^{-1})^{T})$. Find det B.

[7 points]

3. (b) Show that the matrices, $C = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & -2 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 0 & -6 \\ 1 & 0 & -2 \end{pmatrix}$, are **not** row equivalent. . (Recall that two matrices are row-equivalent if one can be obtained from the other by elementary row operations)

[6 points]

4. Let A and B be two 3 x 3 matrices such that AB = - BA. Show that A or B has no inverse.

[7 points]

PART II. Circle the correct answer for each of the following problems (Problem 5 to Problem 12) <u>IN THE TABLE IN THE FRONT PAGE</u>. [5 points for each correct answer. NO PENALTY].

[40 points]

5. Let A be an n×n matrix. Which one of the following statements is **FALSE:**

   (a) If the reduced echelon form of A is I, then A is invertible
   (b) If the echelon form of A is I, then A is invertible
   (c) If A is not invertible, then the matrix equation AX=B has many solutions
   (d) If the homogeneous matrix equation AX=**0** has only the trivial solution, then A is invertible
   (e) If the reduced echelon form of A is not I, then |A|=0

[5 points]

6. Let A be a square matrix such that $A^3=A$. Then

   (a) A is not invertible
   (b) det(A)= ± 1
   (c) If A is invertible then $A^{-1}=A$
   (d) $A^8=A$
   (e) None of the above

[5 points]

7. Suppose A and B are invertible n x n matrices and they commute (AB =BA), then which one of the following statements is **FALSE:**

$(A+B)^{-1}=A^{-1}+B^{-1}$
$(AB)^2=A^2B^2$.
$A^{-1}B=BA^{-1}$.
$A^{-1}B^{-1}=B^{-1}A^{-1}$

[5 points]

8. If A is a 2x2 symmetric matrix, which one of the following statements is **TRUE:**

(a) A is not invertible.
(b) A^2 is symmetric.
(c) A is row equivalent to I.
(d) $\det(A) \neq 0$
(e) None of the above.

[5 points]

9. let A and B be two given nxn matrices. Which one of the following statements is **TRUE**

(a) If A and B are symmetric then AB is symmetric

(b) If A and B are invertible then AB is invertible

(c) If A and B are idempotent then AB is idempotent

(d) If AB=0 then either A=0 or B=0

(e) none of the above

[5 points]

10. Let A be a matrix of the form

$$A = \begin{pmatrix} a & 1 & 0 \\ 0 & a & -1 \\ 0 & 0 & a \end{pmatrix}.$$

Which **one** of the following statements **is FALSE**?

If $a \neq 0, then\ A\ is\ invertible$
If a=1, then det(A)=1.
If $a = 0, then\ A^3 = 0$
If $\det(A) = 0, then\ A^{20} = A$

[5 points]

11. Let A be an invertible n×n matrix. Which one of the following statements is **FALSE**?

A^t is invertible.

There is only one matrix in row echelon form for A.

AB is invertible for any invertible n×n matrix B.

det (A) ≠0.

The reduced row echelon form of A is I.

[5 points]

12. The value(s) of λ for the system with corresponding augmented matrix

$$\left(\begin{array}{ccc|c} 1 & \lambda-2 & 1 & 3 \\ 0 & \lambda-1 & \lambda-1 & 2 \\ 0 & 0 & 0 & \lambda^2-4 \end{array}\right)$$ to be **consistent** are:

(a) λ=2 only
(b) λ= -2 only
(c) λ ≠-2 and λ≠2
(d) λ=2 or λ= -2
(e) none of the above

[5 points]

PART III. Answer **TRUE** or **FALSE** only (**Question 13 to Question 19)** **IN THE TABLE IN THE FRONT PAGE .** (2 points for each correct answer. **NO Penalty**)

[14 points]

13. If A is a 4 x 4 skew symmetric matrix then det A = 0

14. If A and B are n x n invertible that commute (AB=BA) then A-I and B-I commute.

15. If A is skew symmetric then A^2 is symmetric.

16. Let A be an n×n such that $A=A^2$. Then $(I-2A)^{-1} = I - 2A$.

17. If A and B are n x n non-zero matrices such that AB = 0 then A and B are not invertible.

18. If A is a 2 x 2 matrix satisfying det(kA) +det(A) = 0, then A is not invertible.

19. If A is an n x n matrix satisfying $A^2 - 3A + 2I = 0$ then det A= 0.